sciendo

studia humana
QUARTERLY JOURNAL

Studia Humana
Volume 0:0 (2023), pp. 1—5
DOI:

Logic and Metalogic: a Historical Sketch

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Abstract:
This paper briefly discusses the relations between logic and metalogic in history. Metalogic is understood as a reflection on logic in its various senses, particularly *sensu stricto* (formal, mathematical) and *sensu largo* (formal logic plus semantic plus methodology of science). It is shown that metalogic in its contemporary understanding arose after mathematical logic had become a mature discipline. Special passage is devoted to metalogic in Poland. The last part of the paper discussed so-called logocentric predicament.
Keywords: metaphysics, metamathematics, logic *sensu largo*, logic *sensu stricto*, philosophy.

Six words with "meta" at the front have a philosophical significance as related to some specific fields of research. They are: metaphysics, metaphilosophy, metaethics, metascience, metalogic and metamathematics. Doubtless, the first is the most popular – it refers to study of being qua being, one of the most respectable philosophical problems; roughly speaking (but I omit various conceptual and historical issues) the subject of metaphysics can be identified with the scope of ontological investigations. It was probably Andronicos of Rhodes, the scholar of the Peripatetic philosophical school (the Lyceum) in the second half of the 1st B. C., who introduced the phrase *Τὰ μετὰ τὰ φυσικά* (*ta meta ta physika*; what comes after physics – the book called *Physika* was the first item in Andronicos' catalogue of Aritotle's books) as the title for Aristotle's books devoted to the first philosophy (other characterizations include theology, wisdom or the science investigating the first causes of things). Thus, the Greek counterpart of the word "metaphysics" arose after some simplification of the classificatory Andronicos' locution, that is, in a rather accidental way. Medieval philosophers translated *Metaphysika* as *Metaphysica* – this word was employed as the title of Aristotle's book on *prote filosofia* (just the first philosophy). However, the word *metaphysica as* well as its various counterparts in other languages, like German *Metaphysik* or Polish *metafizyka,* lost its meaning as related to ordering the Stagirite writings and began to denote the part of philosophy investigating being, its kinds and properties.

The above mentioned terminological circumstances blocked using the word "metaphysics" as referring to a theory of physics, directed to one of particular sciences. According to the most common view, metaphysics studies the world and its objectual furniture. Other above listed meta-nouns refer

to considerations on something (a domain) to which the word standing just after the prefix "meta" refers. Consequently, metaphilosophy is about philosophy, metaethics about ethics, metascience about science, metalogic about logic and metamathematics about mathematics. We can eventually distinguish first-order disciplines (FOD, for brevity) and second-order ones (SOD, for brevity) – the latter are about of the former. Consequently, philosophy, ethics, science, logic and mathematics are first-order, but metaphilosophy, metaethics, metascience, metalogic and metamathematics – second-order. If we consider FOD as theoretical (I abstract here from entering into a definition of the word "theory), we can distinguish between the theoretical level (that is FOD) and the metatheoretical level as identified with SOD – theories are about the world, but metatheories about theories. Still another aspect appears in saying FOD are expressed in the object language, but metatheories in the metalanguage.

There appear various problems concerning relations between FOD and SOD. In particular, one can ask whether the methodological status is meta-disciplines is the same as disciplines related to them. Is metaphilosophy, a part of philosophy, metaethics – of ethics, metascience – of science, metalogic – of logic and metamathematics – of mathematics. Clearly, it requires further conceptual elaborations concerning all mentioned fields. For instance, logical empiricist defined science as a set of sentences satisfying some epistemic constraints, like the principle of testability, but metascience investigate syntactic and (in the later account of this movement) semantic properties of scientific locutions. However, other approach considers science as constituted by activities of scientists in the academic sense, and metascience as logical, sociological and psychological studies on science – the former belong to FOD, but the latter to SOD. Furthermore, so-called normative ethics aims to formulate ethical norms and evaluations – some authors consider ethical scientific theories to be possible, others deny such a possibility. However, both sides agree that metaethics is fairly legitimate in which concepts employed in normative ethics are analyzed. Perhaps the most dramatic situation occurs in philosophy. Metaphilosophy is certainly considered as a part of philosophy. According to Wittgenstein [see 18., 4.111].

Philosophy is not one of the natural sciences. (The word "philosophy" must mean something which above or below, but not beside the natural sciences).

Husserl and his followers treat philosophy as a super-science staying above natural science, but thinkers, like logical empiricists, consider philosophy as located below mathematics, physics or sociology. Yet philosophical reflection on philosophy itself is accounted as a part of the latter by both parties, identified by Wittgenstein as seeing philosophy as being (staying) above or below natural sciences. Logical empiricists, directly inspired by Wittgenstein's quoted metaphilosophical remarks, accused the traditional philosophy as mostly meaningless (= unscientific) metaphysics. This pejorative qualification of metaphysics has ancestors in Hume and Kant, although the borderline between science and metaphysics was (and still is) drawn differently in each case. Anyway, the problem of how FOD is related to that denoted by the acronym SOD is important in each specific case.

Aristotle's syllogistic was the first fully developed logical theory. Various metalogical rules, for instance, that a correct syllogism must have at least general premise, supplemented theorems of this system. The Stagirite also offered a theory of non-deductive inferences and commented on the question of their value in accommodating truths about the world. He elaborated various philosophical problems, for instance, a general definition of truth and its application to future-contingents. Although Aristotle did not speak about logic *sensu stricto* (in the narrow sense; formal logic) and logic *sensu largo* (in the wide sense; semantics plus formal logic plus methodology of science), this distinction is present in his writings, similarly to the Stoics. Medieval logicians worked in all domains of logic *sensu largo*. John of Salisbury prepared the book *Metalogicon*, but it was rather a textbook of practical logic and its role in human thinking; consequently, he cannot be considered as an anticipant of metalogic in the contemporary sense. Petrus Hispanus attributed the universality property to logic saying that *dialectica* (that is, logic) *est art atrium et scientia scientiarum ad omnium aliarum scientiarum methodorum principia viam habent* (logic is science of sciences, which provides the methodological principles for all other sciences). Theory of *consequentiae* and *suppositiones* or

Occham's nominalism illustrate metalogical themes in the Middle Ages, Leibniz and his ideas of *calculus rationaticator* and *characteristica universalis* can be considered as further examples of logico-metalogical considerations, being an anticipation of so-called *logica magna* (grand logic), a system covering at least the entire mathematics, if not knowledge at all. Kant's account of logic (I abstract from his later idea of transcendental logic) as analytic was a methodological view with an explicit philosophical flavour. Fichte and Bolzano tried to develop *Wissenschaftslehre* (theory of science) with formal logic as its part. The word "metalogic" as referring to logical systems, their nature, their properties, relations to other fields, etc. began to be used in the 19th century, mostly by philosophers from the Neo-Kantian School (see [11]; many historical facts are noted in [5]). Some strange uses occurred as well, for example, Ernest Troeltsch, a distinguished German historian, referred this word to methods of concrete historical investigations as metalogical, and Walter Harburger, a German composer and musicologist, the author of the book *Die Metalogik* (1919), was speaking about metalogic as the logic of music. Yet such usages became forgotten in the course of time.

Mathematics appeared as a separate science in ancient Greece even before logic, namely not later in the Pythagorean School, and very soon achieved a remarkable stage of development, culminating in the antiquity in works of Euclid, Archimedes and Claudius Ptolemy. Such considerations as treating numbers as the *ache* of the reality, seeing geometry as the basic mathematical theory, the invention of deductive method, Plato's account of mathematical objects as abstracts or investigating the relations of the fifth axioms of Euclid to other postulates can be taken as examples of ancient metamathematics (see [13] as a brief presenting more facts, also from the subsequent history). Medieval metamathemical reflection did not develope very much, due to very poor development of mathematics itself. The situation changes in the 16th century and later, of course, after the new mathematical discoveries of Descartes (analytic geometry), Newton (calculus) and Leibniz (also calculus). Berkeley's critique of the concept of infinisitemals was a philosophical-metamathematical analysis. Kant's view that mathematics is, contrary to logic, synthetic a priori, Attempts to prove the parallel axiom from other geometrical assumptions motivated meta-geometry as the first systematic metamathematical theory. However, some mathematicians, even very eminent, like Gauss, strongly protested against the word "metamathematics" as suggesting metaphysical speculations to be avoided by the real sciences. The construction of models for Non-Euclidean geometry (Beltrami, Riemann) convinced mathematicians that metamathematical reflection on mathematics is fruitful and should be continued. This stage was concluded by the rise of set theory (Cantor), program of arithmetization of analysis (Dedekind, Weiestrass) and the axiomatization of geometry (Hilbers) as well as arithmetic of natural numbers (Dedekind, Peano). Mathematical logic developed concurrently to the mentioned novelties in mathematics, firstly as algebra of logic (Boole, Schröder) and secondly as consisting of propositional calculus and quantification (predicate) logic (Frege, Russell) as axiomatic systems.

Since formal properties, like axiomatization, completeness, consistency or independence of axioms, appeared to be essential, this immediately directed logicians' attention to metalogical issues. Three other circumstances strengthened interests in metalogic and metamathematics in 1900-1939. Firstly, logical antinomies had to be solved, which needed various subtle logical investigations, for instance, concerning logical types. Secondly, three leading programs in the foundations of mathematics, namely logicism (a reduction of mathematics to logic), formalism and intuitionism, required a logical elaboration. In the case of logicism (Frege, Russell), the relation of logic and set theory was crucial (resulting systems might be considered as *logica magna* – Leśniewski's logic belongs to this group as well), in the case of formalism (more precisely in the version of this project as presented by Hilbert), the explanation of the scope of finitary methods, and in the case of intuitionism – the logic of constructive methods in mathematics. Hilbert's program inspired metamathematics (metalogic was understood as a part of metamathematical research) much more than other mentioned views, because it claimed that mathematical systems should be investigated by explicitly formulated formal means. Consequently, metalogic became a part of metamathematics (Gödel and Tarski worked within these frameworks). For logicism, the former was still a mixture of

mathematics and philosophy. Brouwer was not interested in logic very much – his foundational project was based on some very speculative philosophical ideas about time-intuition. Intuitionistic logic achieved a mature shape in Heyting's hands in 1930s. Thirdly, various alternative formal logical systems (logic as formalized by Frege and Russell was identified as classical) were proposed in the period in question. C. I. Lewis offered systems of modal logic (or based on strict implication), Łukasiewicz and Post constructed many-valued logic and (see above) intuitionistic logic arose as a device of formalization of intuitionistic mathematics. The plurality of logics generated several metalogical problems, like comparisons of proposed schemes or the question, if any of them is correct in a sense. Łukasiewicz argued that, at least in the case of many-valued logic, the difference between it and two-valued (classical) logic does not concern this or that theorem, in particular, the law of excluded middle, but the principle of bivalence, that is, a fundamental metalogical principle.

Metalogic flourished in Poland. We read [9, pp. 38, 59]:

In the course of the years 1926-1930 investigations were carried out in Warsaw belonging to that part of metamathematics – or better metalogic – which has as its field of study the simplest deductive discipline, namely the sentential calculus. These investigations were initiated by Łukasiewicz; the first results originated both with him and with Tarski. In the seminar for mathematical logic, which was conducted by Łukasiewicz in the University of Warsaw beginning in 1926, most of the results stated below of Lindenbaum, Sobociński, and Wajsberg were found and discussed. The systematization of all the research and the clarification of concepts concerned was the work of Tarski.[,,,]. In conclusion we would like to add that, as the simplest deductive discipline, the sentential calculus is particularly suitable metamathematical investigation. It is to be regarded as a laboratory in which metamathematical methods can be discovered and metamathematical concepts constructed which can then be carried over to more complicated mathematical systems.

Simultaneously, Tarski papers on metamathematics (see [15], [16]) appeared. One can find in these writings investigations on various metamathematical concepts and problems, like deductive system, consequence operation or logical matrix. These results established to a great respect the position of metamathematics and metalogic in mathematical community. It is perhaps worth noting that Polish logicians did not assume any particular system of the foundations of mathematics. Following the tradition of Polish Mathematical School they admitted any accepted mathematical method in order to carry out investigations on logical and mathematical systems – a free use of a controversial axiom of choice is a good example of this attitude. In other words, "Polish" metamathematics was not logicist, formalist or intuitionistc as well as not bounded by any general philosophical view as it occurred in the Vienna Circle syntactic approach (see [6]).

In the last part of the present paper I would like to make some remarks about a problem from the borderline of metalogic and philosophy. Sheffer observed the following situation (see [6, p. 218]) "In order to give an account of logic, we must presuppose and employ logic." He called this dependence "the logocentric predicament." Clearly, it concerns the relation between FOD and SOD as restricted to logic itself. The difficulty consists in the fact that we can either suffer from an regressum ad infinitium or vicious circle. Assume that **L** is logic, which is analysed in metalogic, that is, $\mathbf{L}^{\mathbf{M}}$. In order to explain the validity of **L** (more exactly, its theorem), we need to use logical rule in $\mathbf{L}^{\mathbf{M}}$. However, in order to do that, we must either go to $\mathbf{L}^{\mathbf{MM}}$ (the third level) or to fall into vicious circle. Since the latter outcome is not good, we return to the former, but in consequence, we need to step into $\mathbf{L}^{\mathbf{MMM}}$ and so on. How to resolve this dilemma? Ajdukiewicz (see 1.) proposed the following solution of a dilemma stated by the Sceptics. According to this philosophical truth, any correct truth-criterion **C** is problematic, because in order to use **C**, one needs a criterion, let denote it by $\mathbf{C}^{\mathbf{C}}$, by that **C** is good. Yet $\mathbf{C}^{\mathbf{C}}$ either leads to the infinite regress or is to be blamed for circularity. According to Ajdukiewicz, it is enough to use, but it is unnecessary to know that **C** is good. This idea as applied to the logocentric predicament suggests that it is enough to apply logical rules without knowledge that they are logically valid. This solution can be supplemented by the following observation. We can (see [19]) define logic as universally valid, that is, true (correct) in all possible models (world). Consequently, **L** is also valid in $\mathbf{L}^{\mathbf{M}}$. If one observes that the universality property is defined in $\mathbf{L}^{\mathbf{M}}$ and uses **L,** which is problematic without its grounding in $\mathbf{L}^{\mathbf{M}}$, we can observe that logic is used before

being grounded, but this situation does not generate any theoretical objection. For instance, we can say that logic is genetically inborn in our mental capacities. This example shows a connection between metalogic and fundamental epistemological problems.

A Final Remark

This paper does not claim to be, even approximately, an exhausted treatment of metalogic and its history. See, for instance [3], [4], [7] or [8] for detailed treatments.

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